1. Let $f(x) = \sqrt{x^3}$. Find

(a) $f'(x)$;

(b) $\int f(x)\mathrm{d}x$.

Working:

Answers:

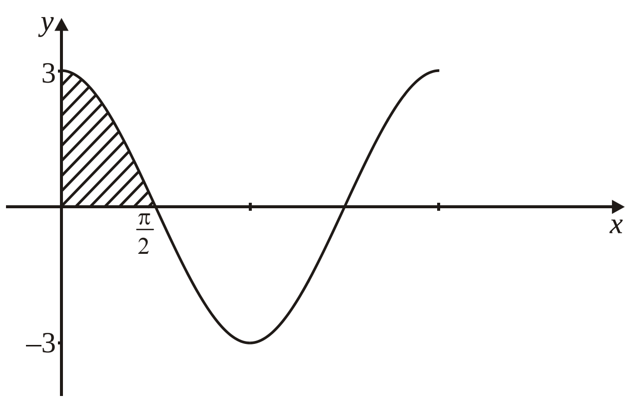
(a) ....

(b) ....

(Total 6 marks)

2. The graph represents the function

$$f : x \mapsto p\cos x,\ p \in \mathbb{N}.$$



Find

(a) the value of p;

(b) the area of the shaded region.

Working:

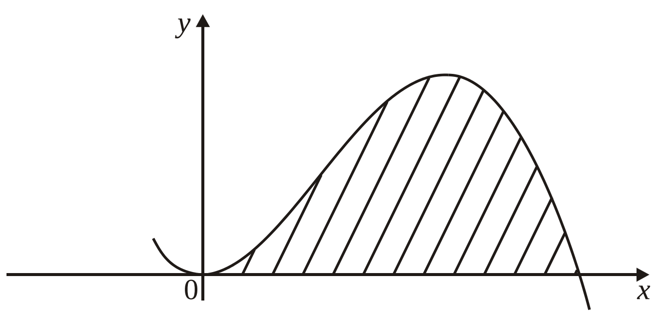
Answers:

(a) ....

(b) ....

(Total 4 marks)

3. The diagram shows part of the graph of $y = 12x^2(1 - x)$.



(a) Write down an integral which represents the area of the shaded region.

(b) Find the area of the shaded region.

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)

4. An aircraft lands on a runway. Its velocity v m s^{-1} at time t seconds after landing is given by the equation $v = 50 + 50e^{-0.5t}$, where $0 \leq t \leq 4$.

(a) Find the velocity of the aircraft

(i) when it lands;

(ii) when $t = 4$.

(4)

(b) Write down an integral which represents the distance travelled in the first four seconds.

(3)

(c) Calculate the distance travelled in the first four seconds.

(2)

After four seconds, the aircraft slows down (decelerates) **at a constant rate** and comes to rest when $t = 11$.

(d) **Sketch** a graph of velocity against time for $0 \leq t \leq 11$. Clearly label the axes and mark on the graph the point where $t = 4$.

(5)

(e) Find the constant rate at which the aircraft is slowing down (decelerating) between $t = 4$ and $t = 11$.

(2)

(f) Calculate the distance travelled by the aircraft between $t = 4$ and $t = 11$.

(2)
(Total 18 marks)

5. Given that $\int_1^3 g(x)\mathrm{d}x = 10$, deduce the value of

(a) $\int_1^3 \frac{1}{2} g(x)\mathrm{d}x$;

(b) $\int_1^3 (g(x)+4)\mathrm{d}x$.

Working:

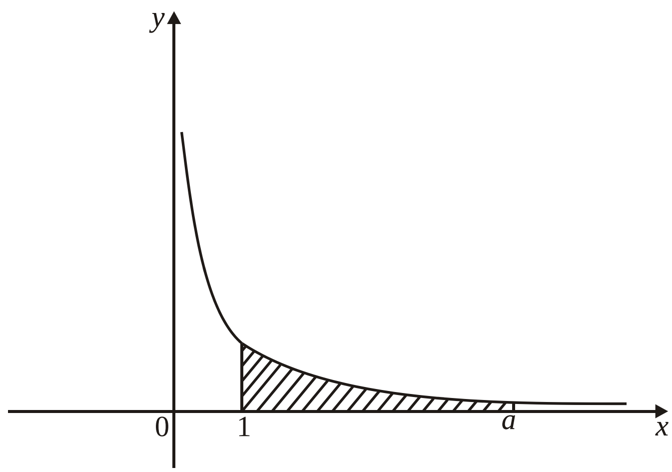
Answers:

(a) ....

(b) ....

(Total 6 marks)

6. The diagram shows part of the graph of $y = \dfrac{1}{x}$. The area of the shaded region is 2 units.



Find the exact value of *a*.

Working:

Answer:

....

(Total 4 marks)

7. A curve with equation $y = f(x)$ passes through the point (1, 1). Its gradient function is $f'(x) = -2x + 3$.

Find the equation of the curve.

Working:

Answer:

....

(Total 4 marks)

8. Find

(a) $\int \sin(3x + 7)\mathrm{d}x$;

(b) $\int e^{-4x}\mathrm{d}x$.

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)

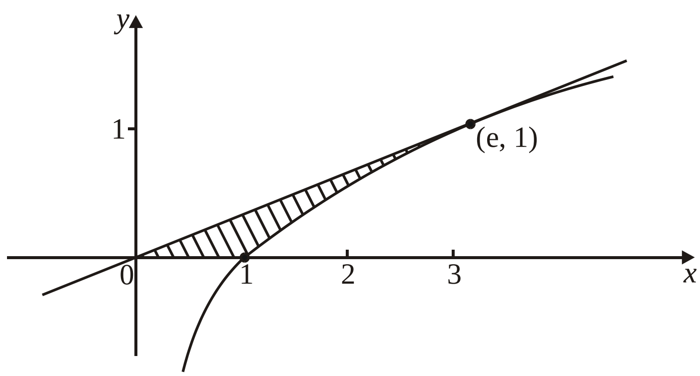
9. (a) Find the equation of the tangent line to the curve $y = \ln x$ at the point (e, 1), and verify that the origin is on this line.

(4)

(b) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x \ln x - x) = \ln x$.

(2)

(c) The diagram shows the region enclosed by the curve $y = \ln x$, the tangent line in part (a), and the line $y = 0$.



Use the result of part (b) to show that the area of this region is $\frac{1}{2}\mathrm{e} - 1$.

(4)

(Total 10 marks)

10. Given that $f(x) = (2x + 5)^3$ find

(a) $f'(x)$;

(b) $\int f(x)\mathrm{d}x$.

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)

11. Let $f'(x) = 1 - x^2$. Given that $f(3) = 0$, find $f(x)$.

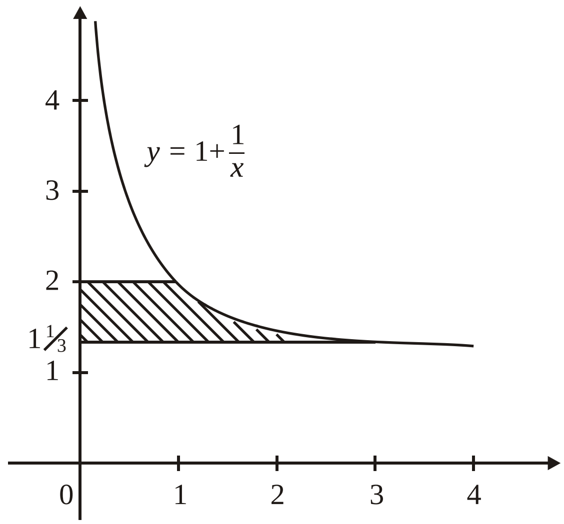
Working:

Answer:

....

(Total 4 marks)

12. The diagram shows the graph of the function $y = 1 + \frac{1}{x}$, $0 < x \leq 4$. Find the **exact** value of the area of the shaded region.



Working:

Answer:

....

(Total 4 marks)

13. **In this question you should note that radians are used throughout.**

(a) (i) Sketch the graph of $y = x^2 \cos x$, for $0 \leq x \leq 2$ making clear the approximate positions of the positive x-intercept, the maximum point and the end-points.

(ii) Write down the **approximate** coordinates of the positive x-intercept, the maximum point and the end-points.

(7)

(b) Find the **exact value** of the positive x-intercept for $0 \leq x \leq 2$.

(2)

Let R be the region in the first quadrant enclosed by the graph and the x-axis.

(c) (i) Shade R on your diagram.

(ii) Write down an integral which represents the area of R.

(3)

(d) Evaluate the integral in part (c)(ii), either by using a graphic display calculator, or by using the following information.

$$\frac{\mathrm{d}}{\mathrm{d}x}(x^2 \sin x + 2x \cos x - 2 \sin x) = x^2 \cos x.$$

(3)
(Total 15 marks)

14. If $f'(x) = \cos x$, and $f\left(\frac{\pi}{2}\right) = -2$, find $f(x)$.

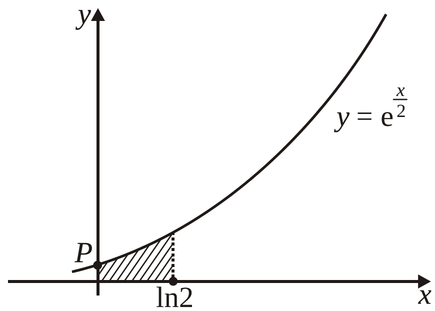
Working:

Answer:

....

(Total 4 marks)

15. The diagram shows part of the graph of $y = e^{\frac{x}{2}}$.



(a) Find the coordinates of the point P, where the graph meets the y-axis.

(2)

The shaded region between the graph and the x-axis, bounded by $x = 0$ and $x = \ln 2$, is rotated through 360° about the x-axis.

(b) Write down an integral which represents the volume of the solid obtained.

(4)

(c) Show that this volume is π.

(5)

(Total 11 marks)

16. In this question, s represents displacement in metres, and t represents time in seconds.

(a) The velocity v m s^{-1} of a moving body may be written as $v = \dfrac{ds}{dt} = 30 - at$, where a is a constant. Given that $s = 0$ when $t = 0$, find an expression for s in terms of a and t.

(5)

Trains approaching a station start to slow down when they pass a signal which is 200 m from the station.

(b) The velocity of Train 1 t seconds after passing the signal is given by $v = 30 - 5t$.

(i) Write down its velocity as it passes the signal.

(ii) Show that it will stop before reaching the station.

(5)

(c) Train 2 slows down so that it stops at the station. Its velocity is given by

$v = \frac{\mathrm{d}s}{\mathrm{d}t} = 30 - at$, where a is a constant.

(i) Find, in terms of a, the time taken to stop.

(ii) Use your solutions to parts (a) and (c)(i) to find the value of a.

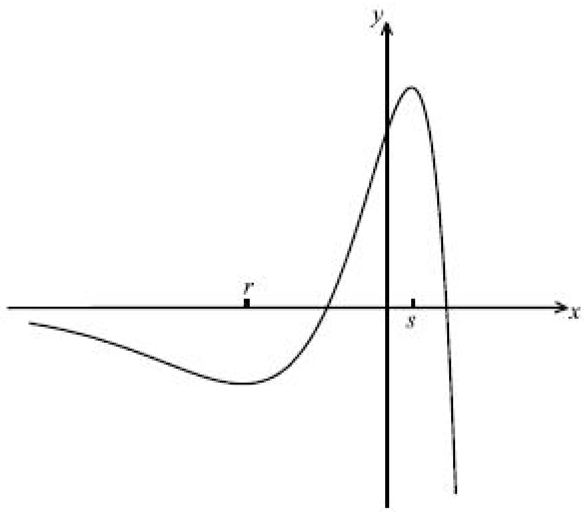
(5)
(Total 15 marks)

17. Let $f(x) = e^x(1 - x^2)$.

(a) Show that $f'(x) = e^x(1 - 2x - x^2)$.

(3)

Part of the graph of $y = f(x)$, for $-6 \le x \le 2$, is shown below. The x-coordinates of the local minimum and maximum points are r and s respectively.



(b) Write down the **equation** of the horizontal asymptote.

(1)

(c) Write down the value of r and of s.

(4)

(d) Let L be the normal to the curve of f at P(0, 1). Show that L has equation $x + y = 1$.

(4)

(e) Let R be the region enclosed by the curve $y = f(x)$ and the line L.

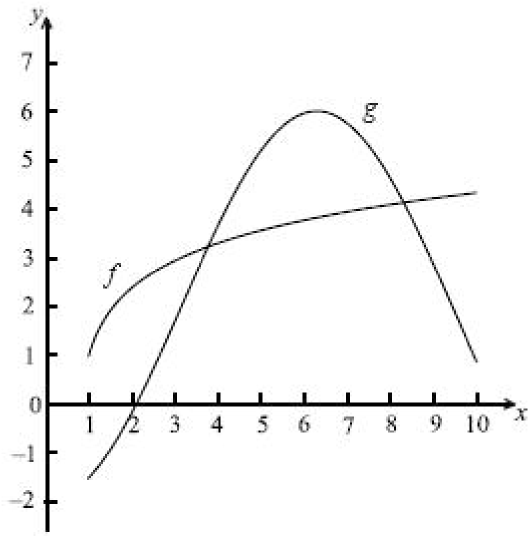
(i) Find an expression for the area of R.

(ii) Calculate the area of R.

(5)

(Total 17 marks)

18. The following diagram shows the graphs of $f(x) = \ln(3x - 2) + 1$ and $g(x) = -4\cos(0.5x) + 2$, for $1 \le x \le 10$.



(a) Let A be the area of the region **enclosed** by the curves of f and g.

(i) Find an expression for A.

(ii) Calculate the value of A.

(6)

(b) (i) Find $f'(x)$.

(ii) Find $g'(x)$.

(4)

(c) There are two values of x for which the gradient of f is equal to the gradient of g. Find both these values of x.

(4)

(Total 14 marks)